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Shishi-odoshi and Large Deviations in Flow Rates per Unit Time

鹿威しで計測する流量率の大きな揺らぎ

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Abstract

The *shishi-odoshi* is a traditional device found in Japanese gardens. It is composed of a bamboo tube that when filled with water swings downward to empty itself and makes a clacking sound as a result. The water-filled bamboo tube clacks against a stone when emptied, and the clacking sound scares beasts and birds away from gardens. For a fluctuating flow rate, intervals between the clacks vary. The flow rate per unit time and the distribution function of the clacking interval can be identified, respectively, as the velocity of a random walker and a first passage time distribution. The rate function for the flow rate per unit time is derived not according to its definition but by use of the distribution function of a first passage time. This idea is illustrated by coin-tossing and large-deviation statistics.

Key words: *Shishi-odoshi*, Flow rate, Random walk, First passage time distribution, Large deviation, Rate function

要旨

鹿威しの振動間隔は，流れ込む水の流量率が一定でないと，揺らぐ．流量率を酔歩粒子の速度とみなし，鹿威しの水受けが満水になるまでの時間を酔歩粒子の初通過時間とみなす．これにより，流量率のレート関数を，流量率の瞬間値とその粗視量の分布を必要とする定義に従った計算ではなく，上述の初通過時間分布から間接的に求めることができる．コイン投げの大偏差統計に従うような流量率を例題として取り上げ，この手法を説明する．

重要語句：鹿威し、流量率、酔歩、初通過時間分布、大偏差統計、レート関数

Introduction

A *shishi-odoshi* consists of a segmented tube, usually made of bamboo, pivoted to one side of its balance point. At rest, its heavier end is lowered and rests against a rock. Water trickling into the upper end of the tube accumulates and eventually moves the tube's center of gravity past the pivot, causing the tube to rotate and dump out the water. The heavier end then falls back against the rock, making a sharp sound, and the cycle repeats. This noise is intended to startle any herbivores such as deer or boars which may be grazing on plants in the garden. Examples of *shishi-odoshi* are illustrated in Ref. (1). In Japan, its sounds are enjoyed in a traditional garden.

In this paper, we assume that the flow rate or the amount of water pouring into a *shishi-odoshi* per unit time fluctuates. Additionally, we discuss the relationship of the distribution of time intervals between clacks to large deviations in the flow rate per unit time.

Formalism

Let V be the volume of a *shishi-odoshi's* water container. The time-dependent flow rate per unit time is denoted as $f(t)$. At $t=t_0$ we start to pour water into the *shishi-odoshi*, and it becomes full at $t=t_0+n$. In this case, the relation $\int_{t_0}^{t_0+n} f(t)\mathrm{d}x = V$ is satisfied, in which n is an interval between the clacks of the *shishi-odoshi*. In the following, an ideal *shishi-odoshi* is considered, which instantaneously discharges the total volume of water when it is full. One may regard f, V and n, respectively, as the velocity of a random walker starting from the origin, a distant goal, and the first passage time when the goal is reached. Thus, by measuring the intervals between the clacks of the *shishi-odoshi*, we can construct a distribution of the first passage time.

The local average z of the flow rate per unit time is given by

$$z = \frac{\int_{t_0}^{t_0+n} f(t)dx}{n} = \frac{V}{n}.$$

The first passage times n vary, and following from the above relation, so do the local averages z. The distribution of z depending on n is denoted as $P(n,z)$, from which we can obtain large deviation statistics for the flow rate per unit time. If n is much larger than its average auto-correlation time of $f(t)$, $P(n,z)$ scales as $P(n,z) = P(n,\overline{z})\exp[-n\psi(z)]$, in which $P(n,\overline{z})$ is an algebraic factor depending on n and $\psi(z)$ is the rate function of the flow rate per unit time (2). Let $\overline{z}$ be the long-time average of z. The rate function is concave, which satisfies $\psi(z)|_{z=\overline{z}} = \frac{d\psi(z)}{dz}\Big|_{z=\overline{z}} = 0$. As a consequence of the central limit theorem, the rate function is quadratic around $z=\overline{z}$.

In our novel viewpoint inspired by the *shishi-odoshi*, we observe directly not the local average z or its instantaneous value of the flow rate per unit time but the first passage time n corresponding to the time interval between clacks of the *shishi-odoshi*. The distribution $P(n,z)$ of z can be regarded as a distribution $Q(V,n)$ of n via the relation $z = V/n$.

The transformation of variable from z to $V = nz$ satisfies the conservation of probability $P(n,z)dz = Q(V,n)dV$, so that we have

$$P(n,z) = Q(V,n)\frac{dV}{dz} = nQ(V,n),$$

$$P(n,\overline{z}) = \overline{n}Q(V,\overline{n}),$$

and the rate function $\psi(z)$ can be indirectly estimated as

$$-\frac{1}{n}\log\frac{nQ(V,n)}{\overline{n}Q(V,\overline{n})}$$

plotted against $z = V/n$, where $\overline{n} = V/\overline{z}$ is the long-time average of the first passage time.

Discussion

Here we provide a discussion based on a concrete example. An event where a water drop either falls or does not fall is assumed to occur at regular unit intervals with equal probability, which is akin to tossing a fair coin. In this case, f is a binary variable with a value of either 0 or 1 depending on an integer-valued time step. Note that the water dripping interval in a real dripping faucet is strongly correlated with the volume of successive water drops, which may be called the flow rate in this case (3). The probability $r(n,z)$ that the head appears nz times in n time steps, or equivalently, the probability that a water drop falls nz times in n time steps, yielding the flow rate per unit time $z = \frac{nz}{n}$, is given by $r(n,z) = \frac{{}_nC_{nz}}{2^n} = \frac{n!}{(nz)!(n-nz)!2^n}$, which can be expressed by V instead of z as $r(n,V/n) = \frac{n!}{(V)!(n-V)!2^n}$. The probability $p(n,V/n)$ that the water container with volume V is filled exactly at time step n is given by $p(n,\frac{V}{n}) = \frac{1}{2}r(n-1,\frac{V-1}{n-1}) = p(n,\frac{V}{n}) = \frac{1}{2}r(n-1,\frac{V-1}{n-1}) = \frac{(n-1)!}{(V-1)!(n-V)!2^n} = q(V,n)$. In Fig. 1, the n-dependences of this probability are plotted for V=2, 3 and 10. Note that both $P(n,z)$ and $Q(V,n)$ in the preceding section are probability densities, and that both $r(n,z)$ and $q(V,n)$ in this section are probabilities and not probability densities.

Taking a large-container limit $V \to \infty$, we applied Stirling's formula $\log N! \sim N\log N - N$ to the factorials, so as to obtain the rate function $-\frac{1}{n}\log p(n,\frac{V}{n}) = \psi(z) = z\log z + (1-z)\log(1-z) + \log 2$ with $z = \frac{V}{n}$. At the long-time average $z = \overline{z} = 1/2$, the relations $\psi(z)|_{z=\overline{z}} = \left.\frac{d\psi(z)}{dz}\right|_{z=\overline{z}} = 0$ are satisfied. In the neighborhood of $z = \overline{z}$, $\psi(z)$ is approximated by the parabola $\psi(z) = 2\left(z-\frac{1}{2}\right)^2$, which implies the central limit theorem.

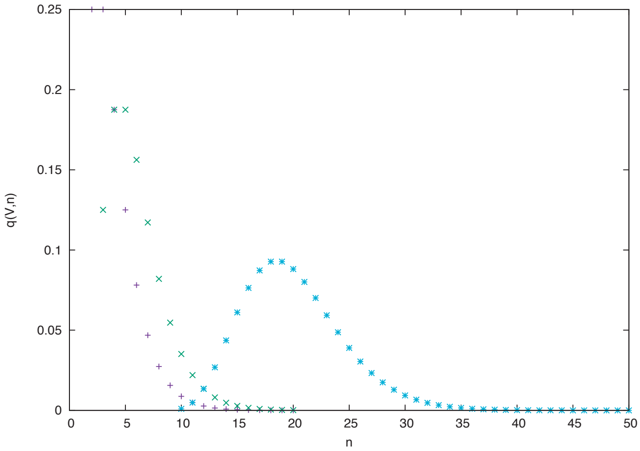

Fig.1. The first passage time distributions $q(V,n)$ plotted against n for $V = 2\ (+),\ 3\ (\times)$ and $10\ (*)$

The rate function of the flow rate per unit time can be estimated as

$$-\frac{1}{n}\log\frac{nq(V,n)}{2Vq(V,2V)}$$

plotted against $z = V/n$, where $\overline{n} = V/\overline{z} = 2V$ is the long-time average of the first passage time, which is shown in Fig. 2 for small-container cases $V = 2\ (+), 3\ (\times)$ and $10\ (*)$ in comparison with the large-container limit (upper line) and the parabola indicating the central limit theorem (lower line). Although the latter only holds around the long-time average, it is extended beyond the applicable range in the figure as a guide to the eye. In spite of small container cases, relatively good agreement is observed with the large-container limit. Even the small-container cases are in agreement with the large-container limit as shown in Fig.2.

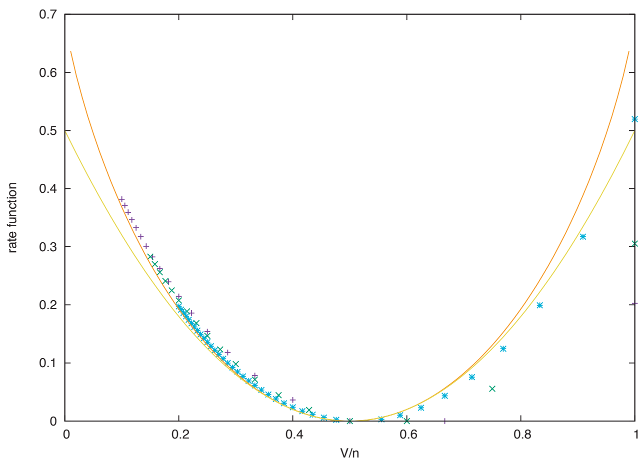

Fig. 2 The approximate rate functions $-\frac{1}{n}\log\frac{nq(V,n)}{2Vq(V,2V)}$ plotted against V/n for $V = 2\ (+),\ 3\ (\times)$ and $10\ (*)$.

The exact rate function (upper line) and the parabola coming from the central limiting theorem (lower line) are also drawn.

Last but certainly not least, our indirect derivation of the rate function from the distribution of the first passage time without observing the instantaneous value $f(t)$ and its local average z can be applied to any stationary fluctuation of $f(t)$, although we confined ourselves to large deviations in the flow rate per unit time inspired by the *shishi-odoshi*. The relation that the sum of a random variable V is equal to the local average z multiplied by the time span n for coarse-graining can be equated to the relation that the distance V is equal to the local average of a random velocity z multiplied by the first passage time n.

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References

1. *Shishi-odoshi*. <https://en.wikipedia.org/wiki/Shishi-odoshi> [accessed on 17 Dec. 2015]
2. Mori, H. & Y. Kuramoto. Dissipative Structures and Chaos, Springer, Heidelberg. (1998).
   Mori, H. & Y. Kuramoto. San'itukozo to kaosu, Iwanami-Shoten, Tokyo (in Japanese). (2000). is also available.
3. Shaw, R. The Dripping Faucet as a Model Chaotic System. Aerial Press, Santa Cruz. (1984).